Unemployment Equilibrium Theory

An intuition from business cycle

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Motivation

Why I studied this?

- This approach was helped by the emergence of new data on job and worker flows at the micro level and the success of the approach in their analysis and explanation.
- The approach proved successful in the modeling of the labor market in equilibrium business cycle models and in the analysis of policy associated with the welfare state.
- Many aspects of the model are too abstract to be empirically implementable as they are presented here.
- The main objective is to develop a usable model that can be extended to deal with many problems in the theory of labor markets and in the empirical analysis of unemployment.
- My "little progress" in my knowledge of macroeconomics.
- This content was extracted from [1]

Benchmark

- Trade in the labor market is a decentralized economic activity:
  - Uncoordinated.
  - Time-Consuming.
  - Costly for both: firms and workers.
  - Firms and workers have to spend resources before job creation and production can take place, and existing jobs command rents in equilibrium.
- I assume a constant level of labor force; trade and production are completely separated activities; a large number of small firms; perfect capital market; only unemployed workers search for work and the equilibrium that I describe is a full rational expectations one.
- I also assume that there is a well-behaved *matching function* that gives the number of jobs formed at any moment in time as a function of the number of workers looking for jobs, the number of firms looking for workers, and possibly some other variables.
- The question here is about the existence of this function.

Trade in the labor market

- Is a nontrivial economic activity
  - Heterogeneity
  - Frictions
  - Information imperfections
- The matching function gives the outcome of the investment of resources by firms and workers in the trading process as a function of the inputs.
- Oof, an another aggregate function...

Definitions

- L : Labor force.
- μ : Unemployment rate.
- ν : Vacancy rate.
- m : Matching function (is the match rate).

$$mL = m(L\mu, L\nu)$$

- The matching function is assumed increasing in both its arguments, concave, and homogeneous of degree 1.
    - For the charts in the following slides I assume a "Cobb Douglas" function, supported by current literature *(I suppose that Christopher and Pissarides was correct about that; when I read "Several of the recent papers cited are unpublished and some may never be published"... I read "please trust me")*
- The model is specified in continuous time.

cont.

- $\varphi \equiv \frac{\nu}{\mu}$: Labor-Market Tightness (from the firm's view).

$$q(\varphi) = \frac{m(\mu L, \nu L)}{\nu L} \equiv m\left(\frac{1}{\varphi}, 1\right) \qquad f(\varphi) = \frac{m(\mu L, \nu L)}{\mu L} \equiv m(1, \varphi)$$

$$\varphi q(\varphi) = f(\varphi) \qquad q'(\varphi) < 0 \quad f'(\varphi) > 0$$

- $\mathcal{X}_t \sim Poisson(t \times q(\varphi))$
- $\mathcal{Y}_t \sim Poisson(t \times f(\varphi))$
- The flow into unemployment results from job-specific (idiosyncratic) shocks that arrive to occupied jobs at the Poisson rate λ: $\mathcal{W}_t \sim Poisson(t \times \lambda)$

cont.

- The mean time duration of a vacant job is $\frac{1}{q(\varphi)}$, and the mean time duration of unemployment is $\frac{1}{f(\varphi)}$.
- During a short interval of time $\mathrm{d}t$:
  - A vacant job is matched to an unemployed worker with probability $q(\varphi)\mathrm{d}t$.
  - An unemployed worker will find a job with probability $f(\varphi)\mathrm{d}t$.
  - A worker moves from employment to unemployment with exogenous probability $\lambda\mathrm{d}t$
- *In "appendix A" I demonstrate all this.*
- The equilibrium is a triple (μ, φ, w) (w is the wage), so we need three independent conditions to find the equilibrium.
  - Beveridge Curve. (BC)
  - Job Creation. (JC)
  - Wage Equation. (WE)

- The mean number of workers who enter unemployment during a small time interval is $(1-\mu)L \times \lambda \mathrm{d}t$ and the mean number who leave unemployment is $\mu L \times f(\varphi)\mathrm{d}t$. The evolution of mean unemployment is given by the difference between the two flows:

$$\mathrm{d}(L\mu) = (1-\mu)L \times \lambda \mathrm{d}t - \mu L \times f(\varphi)\mathrm{d}t$$
$$\implies \dot{\mu} = (1-\mu)\lambda - \mu f(\varphi)$$

In steady state the mean rate of unemployment is constant, so:

$$(1-\mu)\lambda = \mu f(\varphi)$$
$$\mu = \frac{\lambda}{\lambda + f(\varphi)} \tag{BC}$$

Three keys

Job Creation

- Job creation takes place when a firm and a worker meet and agree to an employment contract.
  - Each firms has one job that is vacant when it first enters the market, but the job is occupied by a worker after an employment contract has been signed.
  - Hours of work are fixed (and normalized to unity), and either side can break the contract at any time
  - The capital decision is not important for the main results.
  - The value of a job's output is some constant $p > 0$.
  - When the job is vacant, the firm is actively engaged in hiring at a fixed cost $pc > 0$ per unit time.
- The number of jobs is endogenous and determined by profit maximization. Any firm is free to open a job vacancy and engage in hiring. Hence profit maximization requires that the profit from one more vacancy should be zero.

- Let J be the present-discounted value of expected profit from an occupied job and V the present-discounted value of expected profit from a vacant job. with the above assumptions V and J satisfy the following conditions

$$rV = q(\varphi) \times (J - V) - pc \tag{NAV}$$
$$rJ = p - w - \lambda \times J \tag{NAJ}$$

- In equilibrium $V = 0$, implying that $J = \frac{pc}{q(\varphi)}$ and:

$$p - w - \frac{(r + \lambda)pc}{q(\varphi)} = 0 \tag{JC}$$

Worker's decision

- Workers normally influence the equilibrium outcome through their job search and their influence on wage determination. In this model the only influence that workers have on the equilibrium outcome is through wages.
- During search the worker enjoys some real return x, which we measure in the same units as real wages.
- Let U and W denote the present-discounted value of the expected income stream of, respectively, an unemployed and an employed worker, including the imputed return from nonmarket activities.

Worker's decision

Non-Arbitration Equations

- Both U and W satisfy the following equations:

$$rU = x + f(\varphi) \times (W - U) \tag{NAU}$$
$$rW = w + \lambda \times (U - W) \tag{NAW}$$

- Solving for rU and rW:

$$rU = \frac{(r + \lambda)x + f(\varphi)w}{r + \lambda + f(\varphi)}$$
$$rW = \frac{\lambda x + [r + f(\varphi)]w}{r + \lambda + f(\varphi)}$$

- $W > U$ if $w > x$.

Three keys

Wage Equation

- An employment contract between the meeting firm and worker is a wage w_i for each period of time that they are together and a separation rule that is contingent on the arrival of an idiosyncratic shock.
- I assume both here and throughout the analysis that the wage contract is renegotiated whenever new information arrives.
- For a wage rate w_i the firm's expected return from the job, J_i, satisfies:

$$rJ_i = p - w - \lambda J_i \qquad \frac{\partial J_i}{\partial w_i} = \frac{-1}{r+\lambda} \tag{1}$$

- The job is worth for the worker

$$rW_i = w_i - \lambda \times (W_i - U) \qquad \frac{\partial W_i}{\partial w_i} = \frac{1}{r+\lambda} \tag{2}$$

Three keys

Wage Equation: Nash Bargaining

- The wage for this job satisfies the following maximization:

$$w_i = \arg\max_{w_i \geq 0} \left\{ (W_i - U)^{\beta} (J_i - V)^{1-\beta} \right\}$$

- FOC:

$$W_i - U = \beta \times (J_i + W_i - V - U) \tag{NB}$$

- Imposing the equilibrium condition $V = 0$ and replacing (1) and (2) into (NB):

$$\frac{w_i + \lambda U}{r + \lambda} - U = \left(\frac{p - w_i}{r + \lambda} + \frac{w_i - \lambda U}{r + \lambda} - U \right)$$

$$w_i = rU + \beta(p - rU)$$

Three keys

Wage Equation

- Since $V = 0$ then $J_i = \frac{pc}{q(\varphi)}$. Rewriting (NB) we have:

$$W_i - U = \frac{\beta}{1-\beta} \times J_i = \frac{\beta}{1-\beta} \times \frac{pc}{q(\varphi)}$$

- By making use of this to substitute $W - U$ out of (NAU):

$$rU = x + f(\varphi) \times \frac{\beta}{1-\beta} \times \frac{pc}{q(\varphi)}$$

- Using rU to substitute in w_i's equation (remember that $\varphi q(\varphi) = f(\varphi)$):

$$w_i = (1-\beta) rU + \beta p$$
$$w = \beta p(1 + c\varphi) + (1-\beta)x \qquad \text{(WE)}$$

Steady State

- Equilibrium is a triple (μ, φ, w) that satisfies the flow equilibrium condition (BC), the job creation condition (JC), and the wage equation (WE). The three keys are as follows:

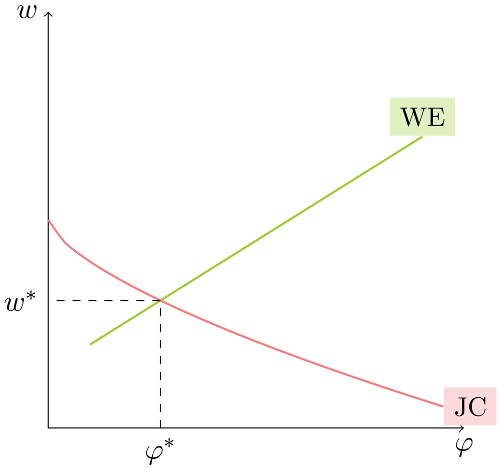
$$\mu = \frac{\lambda}{\lambda + f(\varphi)} \tag{BC}$$

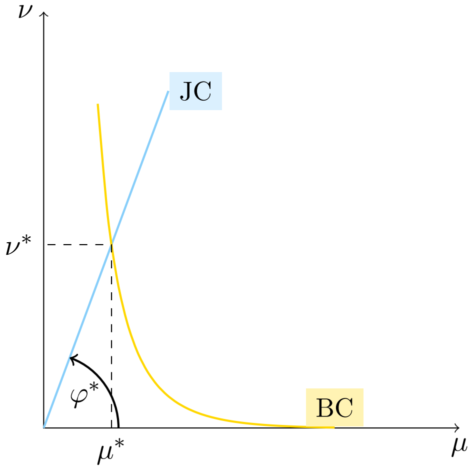
$$p - w - \frac{(r+\lambda)pc}{q(\varphi)} = 0 \tag{JC}$$

$$w = \beta p(1 + c\varphi) + (1-\beta)x \tag{WE}$$

- Equilibrium is easily shown to be unique with the help of two diagrams, one that replaces the conventional demand and supply diagram for labor and a new diagram with the Beveridge curve as its centerpiece.

Diagrams

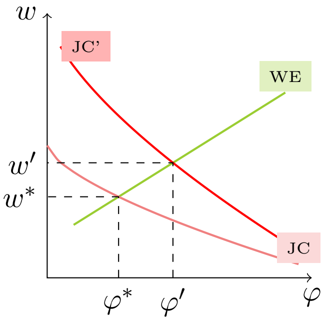


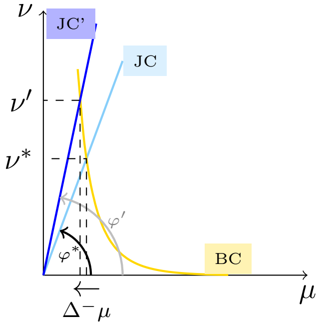


Comparative Statics

$\Delta^+ p$ effect

- Higher labor productivity increases p and shifts the job creation line to the right and the wage curve up. The job creation curve shifts by more, so both wages and market tightness increase. In $\mu \times \nu$ diagram, this rotates the job creation line anticlockwise, increasing vacancies and reducing unemployment.

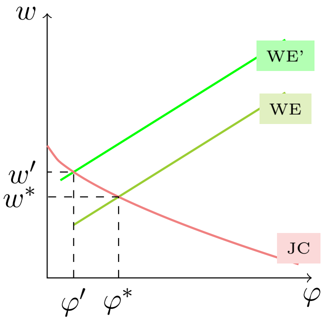


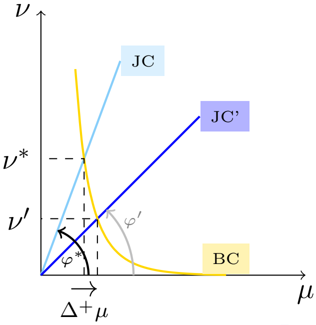


Comparative Statics

$\Delta^+ x$ effect

- A higher x shifts the wage curve up and therefore increases wages but reduces market tightness. Workers claim a higher wage because the cost of unemployment is lower, and with higher wages firms create fewer jobs. The job creation line in $\mu \times \nu$ graphic rotates clockwise, reducing vacancies and increasing unemployment.



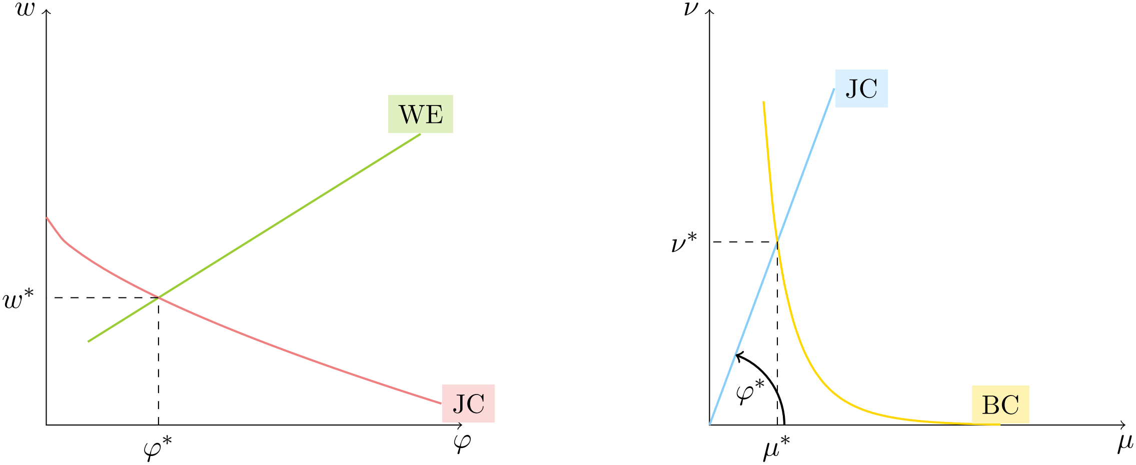


Comparative Statistics

$\Delta^+ r$ and $\Delta^+ \lambda$ effect

- A higher real interest rate or arrival rate of negative idiosyncratic shocks (and so higher job destruction rate at given unemployment rate) shifts the job creation curve in $\varphi \times w$ graphic to the left. The reason is that in the case of the higher interest rate, the future revenues from a job are discounted more heavily, and in the case of the higher destruction rate, the life of the job is on average shorter. They both lead to lower tightness and wage rate because the costs of job creation have to be paid up front. In the Beveridge diagram both changes rotate the job creation line down, but the increase in the arrival rate of idiosyncratic shocks also shifts the Beveridge curve out. The higher interest rate increases unemployment and reduces vacancies. The higher job destruction rate increases unemployment but has uncertain effect on vacancies.

$\Delta^+\lambda$



References

[1] Christopher A. Pissarides. *Equilibrium unemployment theory*. eng. Second edition. Cambridge, Mass: MIT Press, 2000. ISBN: 0262161877.