Underbidding for Oil and Gas Tracts

Julien Martin, Martin Pesendorfer and Jack Shannon

TEC

The Economic Club

Framework

- Bidders: $\mathcal{N} = \{1, \ldots, n\}$
- Tract has a common value $v \in V = [\underline{v}, \overline{v}] \subset \mathbb{R}$.
- Bidder i receives private information $x_i \in [\underline{x}, \overline{x}] = \mathcal{X} \subset \mathbb{R}$.
- Random variables: $\mathbf{X} = (X_1, \ldots, X_n)$; realizations: $\mathbf{x} = (x_1, \ldots, x_n)$.
- Joint distribution: $F_{\mathcal{N}}(\mathbf{x}, v)$. Conditional distribution: $F_{\mathcal{N}}(\mathbf{x}|v)$. Reserve price: r.
- Vector of bids: $\mathbf{b} = (b_1, \ldots, b_n) \in \mathcal{B}^n = [0, \overline{v}]^n$
- Let the set of winners and probability of win be:

$$\begin{aligned} W(\mathbf{b}) &= \{i : (b_i \geq b_j \ \forall j \in \mathcal{N}) \wedge (b_i \geq r)\} \\ Q_i(\mathbf{b}) &= \begin{cases} 0, & b_i \notin W(\mathbf{b}) \\ \frac{1}{|W(\mathbf{b})|}, & b_i \in W(\mathbf{b}) \end{cases} \end{aligned}$$

Framework

- Let player i's strategy: $\beta_i : [\underline{x}, \overline{x}] \to \mathcal{B}$.
- Σ_i: Set of strategies for i.
- Let $\boldsymbol{\beta} \in \times_{i \in \mathcal{N}} \Sigma_i$ denote a strategy profile. Bidder i's ex ante payoff from the first-price auction:

$$u_i(\boldsymbol{\beta}) = \int_{v \in V} \int_{\mathbf{x} \in \mathcal{X}^n} [v - \beta_i(x_i)]\, Q_i\left(\boldsymbol{\beta}(\mathbf{x})\right) F_{\mathcal{N}}(\mathrm{d}\mathbf{x}, \mathrm{d}v)$$

Testable Implications

Independence

$$H_0^{B|X} : (B_i \perp B_j) \mid v, \quad \forall\ i, j \in \mathcal{N}$$

Testable Implications

Best Response Test

- Consider a deviation $\varphi : \mathcal{B} \to \mathcal{B}$. The ex ante payoff under the deviation strategy:

$$u_i((\varphi \circ \beta_i), \boldsymbol{\beta}_{-i}) = \int_{v \in V} \int_{\mathbf{x} \in \mathcal{X}^n} [v - (\varphi \circ \beta_i)(x_i)] \, Q_i \left((\varphi \circ \beta_i)(x_i), \boldsymbol{\beta}_{-i}(\mathbf{x}_{-i}) \right) F_{\mathcal{N}}(\mathrm{d}\mathbf{x}, \mathrm{d}v)$$

- Consider a linear deviation $\varphi(\beta) = \alpha\beta$, and let:

$$\alpha^* = \arg\max_{\alpha \in \mathbb{R}} u_i(\alpha\beta_i, \boldsymbol{\beta}_{-i})$$

- Under BNE, the only deviation must be the identity mapping.

$$H_0 : \alpha^* = 1$$

Testable Implications

Uniform Upward Deviation

- A uniform upward deviation to $\tilde{b} = \beta_i(\tilde{x})$ is defined as a deviation $\tilde{\beta}_i$ from strategy β_i where submitted bids are equal to $\tilde{\beta}_i(x) = \max\{\tilde{b}, \beta_i(x)\}$.
- Assume $X_1, \ldots, X_n | v$ *i.i.d.* with $F_i(x|v) \equiv F(x|v)$ and $F_{-i}(\mathbf{x}_{-i}|v) \equiv \times_{j \neq i} F(x_j|v)$.

$$
\begin{aligned}
u_i(\beta_i, \boldsymbol{\beta}_{-i}) &= \int_{v \in V} \int_{\mathbf{x} \in \mathcal{X}^n} [v - \beta_i(x_i)]\, Q_i(\boldsymbol{\beta}(\mathbf{x})) F_{\mathcal{N}}(\mathrm{d}\mathbf{x}, \mathrm{d}v) \\
&= \int_{v \in V} \int_{x_i \in \mathcal{X}} \int_{\mathbf{x}_{-i} \in \mathcal{X}^{n-1}} [v - \beta_i(x_i)]\, Q_i(\boldsymbol{\beta}(\mathbf{x})) F_{\mathcal{N}}(\mathrm{d}\mathbf{x}|v) G(\mathrm{d}v) \\
&= \int_{v \in V} \int_{x_i \in \mathcal{X}} [v - \beta_i(x_i)] \underbrace{\left(\int_{\mathbf{x}_{-i} \in \mathcal{X}^{n-1}} Q_i(\boldsymbol{\beta}(\mathbf{x})) F_{-i}(\mathrm{d}\mathbf{x}_{-i}|v) \right)}_{q_i(\beta_i)} F(\mathrm{d}x_i|v) G(v)
\end{aligned}
$$

Testable Implications

Uniform Upward Deviation

- Ex ante payoff of bidder i in equilibrium:

$$
\begin{aligned}
u_i(\beta_i, \boldsymbol{\beta}_{-i}) &= \int_{v \in V} \int_{x_i \in \mathcal{X}} [v - \beta_i(x_i)] \underbrace{\left(\int_{\mathbf{x}_{-i} \in \mathcal{X}^{n-1}} Q_i(\boldsymbol{\beta}(\mathbf{x})) F_{-i}(\mathrm{d}\mathbf{x}_{-i} | v) \right)}_{q_i(\beta_i)} F(\mathrm{d}x_i | v) G(\mathrm{d}v) \\
&= \int_{v \in V} \int_{x_i \in \mathcal{X}} [v - \beta_i(x_i)] \, q_i(\beta_i(x_i)) F(\mathrm{d}x_i | v) G(\mathrm{d}v)
\end{aligned}
$$

Testable Implications

Uniform Upward Deviation

- Ex ante payoff of bidder i when he takes a deviation $\tilde{\beta}_i$:

$$
\begin{aligned}
u_i(\tilde{\beta}_i, \boldsymbol{\beta}_{-i}) &= \int_{v \in V} \int_{\underline{x}}^{\tilde{x}} \left(v - \tilde{b}\right) q_i\left(\tilde{b}\right) F(\mathrm{d}x_i|v) G(\mathrm{d}v) \\
&\quad + \int_{v \in V} \int_{\tilde{x}}^{\overline{x}} \left[v - \beta_i(x_i)\right] q_i(\boldsymbol{\beta}(\mathbf{x})) F(\mathrm{d}x_i|v) G(\mathrm{d}v) \\
&= \int_{v \in V} \left(v - \tilde{b}\right) q_i\left(\tilde{b}\right) F(\tilde{x}|v) G(\mathrm{d}v) \\
&\quad + \int_{v \in V} \int_{\tilde{x}}^{\overline{x}} \left[v - \beta_i(x_i)\right] q_i(\boldsymbol{\beta}(\mathbf{x})) F(\mathrm{d}x_i|v) G(\mathrm{d}v)
\end{aligned}
$$

Testable Implications

Uniform Upward Deviation

- A non empty set $S \subset \mathcal{N}$ must have a nonpositive expected net deviation payoff:

$$H_0 :$$

$$\frac{|S| \int_{v \in V} \left(v - \tilde{b}\right) q_i \left(\tilde{b}\right) F(\tilde{x}|v) G(v) - \int_{v \in V} \int_{\underline{x}}^{\tilde{x}} [v - \beta_i(x_i)] \sum_{j \in S} q_j(\beta_j(x_j)) F(\mathrm{d}x_j|v) G(v)}{\int_{v \in V} \int_{\underline{x}}^{\tilde{x}} [v - \beta_i(x_i)] \sum_{j \in S} q_j(\beta_j(x_j)) F(\mathrm{d}x_j|v) G(v)} \leq 0$$

Data

TABLE 1 - SUMMARY STATISTICS FOR AWARDED TRACTS

	All	First-Price	English
Number of auctions	9717	4535	5182
Gross revenue (minus royalty and tax payment)	464.1	521.9	413.6
	(2745)	(3127)	(2360)
Well cost	180.5	183.4	178.1
	(700)	(747)	(656.2)
Net revenue v	283.6	338.5	235.5
	(2197)	(2507)	(1885)
Winning bid	52.82	28.68	47.7
	(113.5)	(138.8)	(85)
Reserve price	4.71	4.53	4.87
	(4.398)	(4.302)	(4.474)
Fraction drilled	0.125	0.119	0.131
	(0.331)	(0.323)	(0.337)
Well cost of drilled tracts	1443	1543	1363
	(1448)	(1613)	(1297)
Fraction productive when drilled	0.885	0.902	0.872
	(0.319)	(0.298)	(0.334)

Data

Table 2 - Bidding Returns for Top Bidders

Bidder	No. of bids	No. of wins	Return v on average	Bid b across	ROI auctions won
Yates Petroleum Corp.	5810	4087	210.46	30.67	1225
Daniel E. Gonzalez	828	592	571.12	65.57	589
Doug J. Schutz	784	548	235.56	68.78	20
The Blanco Comp.	617	103	413.48	14.39	6240
Slash Exploration LP	403	95	22.96	10.94	352
Chase Oil Corporation	320	227	102.17	43.83	534
Featherstone Dev. C.	284	121	305.3	27.58	572
Marbob Energy Corp	278	130	726.32	78.4	698
Bar Cane Inc.	220	129	391.74	60.95	461
Fringe	N/A	3640	342.98	76.02	601

Conditional Independence Test

- z : auction date variable.
- Null Hypothesis:

$$H_0^{B|X} : (B_i \perp B_j) \mid (v, z), \quad \forall i, j \in \mathcal{N}$$

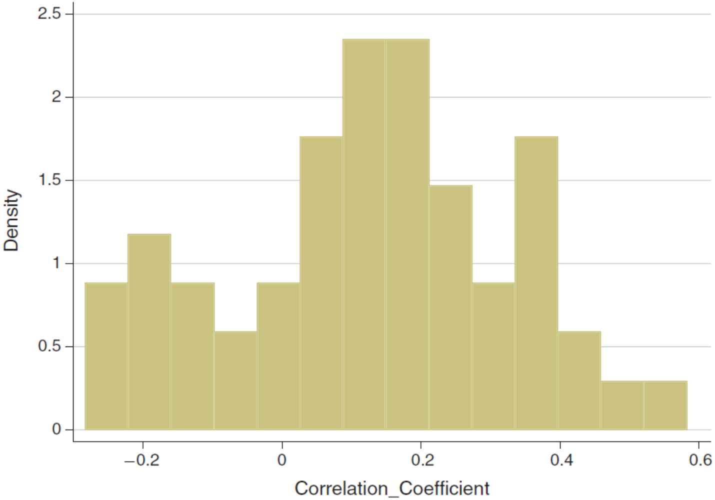
- Alternative hypothesis:

$$H_0^{B|X} : (B_i \not\perp B_j) \mid (v, z), \quad \forall i, j \in \mathcal{N}$$

TABLE 3 - BIPROBIT CORRELATION COEFFICIENTS

Pairs	H_0 of zero correlation rejected at			Sign of correlation coefficient	
	10%	5%	1%	Positive	Negative
55	26	25	14	40	15

Conditional Independence Test



Underbidding

$$H_0 : \alpha^* \equiv \arg\max_{\alpha \in \mathbb{R}} \int_{v \in V} \int_{\mathbf{x} \in \mathcal{X}^n} [v - \alpha\beta_i(x_i)]\, q_i(\alpha\beta_i(x_i), \boldsymbol{\beta}_{-i}(\mathbf{x}_{-i})) F_{\mathcal{N}}(\mathrm{d}\mathbf{x}, \mathrm{d}v) = 1$$

- Let $S \subseteq \mathcal{N}$ denote a subset of bidders, $\alpha \in \mathbb{R}$ a scalar and $\mathcal{T}_i$ a set of auctions in which bidder i participated.
- Test statistic:

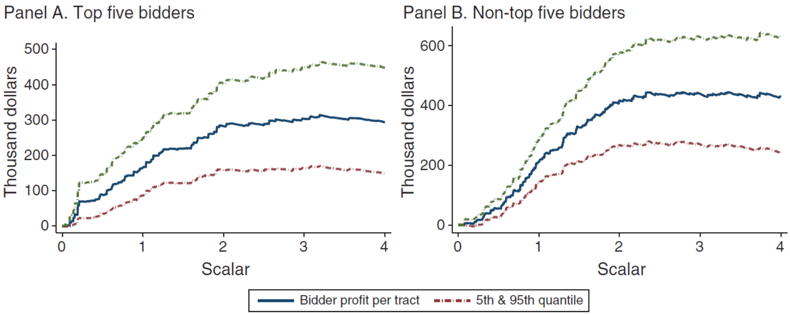
$$\hat{\alpha}^* = \arg\max_{\alpha \in \mathbb{R}} \frac{1}{|S|} \sum_{i \in S} \frac{1}{|\mathcal{T}_i|} \sum_{t \in \mathcal{T}_i} (v_t - \alpha b_{it})\, q_i(\alpha b_{it}, \mathbf{b}_{-i,t})$$

- $\hat{\alpha}^* < 1 \implies$ potential winner's curse or risk aversion
- $\hat{\alpha}^* > 1 \implies$ suggest that bidder i underbid.

Underbidding

Table 4 - Best Response Test: Optimal Bid Scalar $\hat{\alpha}^*$

		Bidder	
	Overall	Top 5	Non-Top 5
Bid scalar estimate $\hat{\alpha}^*$	3.26	3.19	2.78
Fifth and ninety-fifth quantile	[2.19,3.79]	[2.04, 3.19]	[2.15, 3.82]



Uniform Upward Deviation

- Let v_t the observed value, p_t the winning price, p_{it} the winning price when bidder i won the auction and a_t the acreage of the tract.

$$D_T^S = \max_{b \in B} \left\{ \frac{|S| \sum_{t=1}^{T} [v_t - ba_t] \times \mathbf{1}\{ba_t \geq p_t\}}{\sum_{i \in S} \sum_{t=1}^{T} [v_t - p_t] \times \mathbf{1}\{p_{it} = p_t\}} - \frac{\sum_{i \in S} \sum_{t=1}^{T} [v_t - p_t] \times \mathbf{1}\{p_{it} = p_t\} \times \mathbf{1}\{ba_t \geq p_t\}}{\sum_{i \in S} \sum_{t=1}^{T} [v_t - p_t] \times \mathbf{1}\{p_{it} = p_t\}} \right\}$$

$$H_0 : D_T^S \leq 0 \quad H_1 : D_T^S > 0$$

Uniform Upward Deviation

Table 5 - Testing Uniform Upward Deviation Incentives

		All	All	Top five	Top three
Set of bidders when $\lvert\hat{S}\rvert$ equals		13	7	5	3
Baseline estimates	D_T^S	5.29 (1.15)	2.55 (0.58)	4.24 (1)	3.61 (1.51)